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SPECTRAL CHARACTERISTICS OF A FLICKER NOISE MODEL, (U)
JAN 77 G K MANZIE, C I SACH
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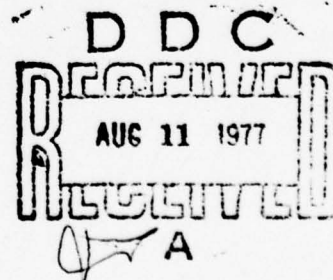
REPORT

MRL-R-680

SPECTRAL CHARACTERISTICS OF A FLICKER
NOISE MODEL

G. K. Manzie and C. I. Sach

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REPORT

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SPECTRAL CHARACTERISTICS OF A FLICKER
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SUMMARY

12 26 p.

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An equation is developed to generate discrete values of "flicker", or $1/f^{1.5}$ noise, for use in mathematical modelling. Spectral analyses of the simulated noise produced by this equation show that the spectral power response varies with the number of samples used in the calculation.

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SPECTRAL CHARACTERISTICS OF A FLICKER NOISE MODEL

1. INTRODUCTION

Various papers have been written on the simulation of "flicker" or " $\frac{1}{f}$ " noise. However, the majority of them are concerned with the effect of flicker noise on frequency measurement and hence investigated flicker noise as a phase variation. We are more interested in simulating flicker noise inherent in the output signals from infrared detectors and their associated electronics.

There are several possible methods of simulating flicker noise. One method is to consider a filter operating on white noise which gives the required spectral variation. This is basically the method Barnes and Allan (1) used in obtaining an expression for variation in phase due to flicker noise. Another system which would seem to have a more reasonable basis as a physical model for flicker noise is the mechanical model of Halford (2) which considers classes of time-dependent perturbations occurring at random, generating random noise with the required spectral density.

2. THEORY

We elected to use the simpler approach of a filter operating on white noise as shown in Figure 1.

We wish to generate noise with the spectral power density, $\phi(f)$, of the form

$$\phi(f) \propto \frac{1}{f^{2\lambda}}, \quad \dots (1)$$

where $\lambda = \frac{1}{2}$ for "flicker" noise.

If we pass white noise, with a constant spectral power density, through a network with a power transfer function proportional to $\frac{1}{f^{2\lambda}}$, then the

output spectrum will have the required characteristics. The network must have a signal transfer function of the form

$$G(f) = \frac{K}{(jf)^\lambda}, \quad \dots (2)$$

where K is a constant. However, a digital computer simulation will generally be working in the time, rather than the frequency domain, so it is necessary to define the processing network G(f) in the time domain.

The impulse response H(t) of G(f) is its Fourier Transform, that is

$$H(t) = \int_{-\infty}^{\infty} G(f) e^{+j2\pi ft} df \quad \dots (3)$$

It is shown in Appendix I that a possible form is

$$H(t) = 2K \sin(\pi\lambda) \Gamma(1-\lambda) (2\pi t)^{\lambda-1}, \quad t > 0 \text{ and } 0 < \lambda < 1 \quad \dots (4)$$

where $\Gamma(\lambda) = \int_0^{\infty} e^{-x} x^{\lambda-1} dx$ for $\lambda > 0$. The output signal y(t) in response to x(t) is normally given by the convolution integral

$$y(t) = \int_{-\infty}^t x(\tau) H(t-\tau) d\tau \quad \dots (5)$$

However, this integral does not exist when x(t) is white noise and the impulse H(t) is of the form given in equation (4). To overcome this problem we will modify the impulse response so that it is non-zero for only a finite duration T and is zero outside this range. Such a change amounts to modifying the power transfer function so that it does not follow an $f^{-2\lambda}$ law down to zero frequency, but introduces a levelling off at low frequencies. The output y(t) will now be expected to have the required spectral law only over a restricted frequency range and as such can be best described as a quasi-flicker noise process. The modified (truncated) impulse will be taken as

$$\left. \begin{aligned} H_M(t) &= 2K \sin(\pi\lambda) \Gamma(1-\lambda) (2\pi t)^{\lambda-1}, \quad 0 < t \leq T \\ H_M(t) &= 0 \quad \text{elsewhere} \end{aligned} \right\} \quad \dots (6)$$

Equations (5) and (6) are applicable to continuous processes. However, for digital computer simulation a discrete process with sampling at regular time intervals Δt must be used. Therefore we consider, instead of the continuous $H_M(t)$, a discrete value impulse response which at time $j\Delta t$ is

$$\bar{H}_M(j\Delta t) = \sum_{i=1}^j h_i \delta(t - i\Delta t), \quad \dots (7)$$

for $j \leq N$ where $N\Delta t = T$, the duration of the impulse response. For $j > N$, $\bar{H}_M(j\Delta t)$ is zero as required by equation (6).

Using (7), and a discrete form of (5), it follows that

$$y_j = y(j\Delta t) = \Delta t \sum_{i=j-N}^{j-1} x_i h_{j-i} \quad \dots (8)$$

If we now consider the particular case of $\lambda = \frac{1}{2}$, then

$$H_M(t) = \sqrt{2} K t^{-\frac{1}{2}} \quad \dots (9)$$

The coefficients h_i introduced in (7), can be defined in two ways. Firstly it can be defined that

$$h_i \Delta t = \int_{i\Delta t}^{(i+1)\Delta t} H_M(u) du, \quad \dots (10)$$

for i integer in the range $0 \leq i \leq N$. This approach leads to

$$h_i = 2\sqrt{2} K (\Delta t)^{-\frac{1}{2}} \left[\sqrt{i+1} - \sqrt{i} \right] \quad \dots (11)$$

Rather than the previous approach, we choose to define

$$h_i = \sqrt{2} K (i\Delta t)^{-\frac{1}{2}} \quad \dots (12)$$

Equation (8) is evaluated using the N coefficients h_1 to h_N defined by (12) to give

$$y_j = \sqrt{2} K(\Delta t)^{\frac{1}{2}} \sum_{i=j-N}^{j-1} \left\{ \frac{x_i}{\sqrt{j-i}} \right\} \quad \dots (13)$$

This equation then uses only the last N values of x_i to generate $y_j(N)$. This then introduces a low frequency cutoff of the $\frac{1}{f}$ behaviour at a frequency of about $\frac{1}{N\Delta t}$. The upper frequency cutoff is at the Nyquist frequency of $\frac{1}{2\Delta t}$.

The problem now arises of investigating how closely the expression in equation (13) produces a spectral power density with a $\frac{1}{f}$ characteristic over the expected band. There are at least two methods of estimating spectral power density.

Firstly, an estimate of the power spectrum can be obtained via the Fast Fourier Transform (3) (FFT). However, as stated by Furber (4) a single FFT calculation from one set of samples of a random signal is a poor estimate of the power spectrum and is a statistically unstable estimate. The stability can be increased by performing FFT calculations on a number (M) of different sets of samples of the stationary signal and then average these together. The variance of the estimate decreases as $\frac{1}{M}$.

The second method is to calculate the autocorrelation function and then calculate the Fourier transform of this function. Since we could calculate an analytical solution for the autocorrelation function we used this second method for calculating the spectral power density.

3. CALCULATION OF AUTOCORRELATION FUNCTION

It is shown by Papoulis (5) that the autocorrelation function of the random signal $y(t)$ generated by applying white noise of uniform spectral power density S to a system with impulse response, $H(t)$, is given by

$$R(\tau) = S \int_0^{\infty} H(u) H(\tau+u) du, \quad \dots (14)$$

where, as usual,

$$R(\tau) = \langle y(t) y(t-\tau) \rangle, \quad \dots (15)$$

where τ is the lag variable and the brackets signify expectation value.

It is readily shown that the discrete form of (14) can, for a finite duration impulse response, be written as

$$R(n, N) = S \Delta t \sum_{j=1}^{N-n} h_j h_{n+j}, \quad \dots (16)$$

Where the delay time τ is given by $n \Delta t$. It should be noted that the above expression was obtained for $0 \leq n \leq N-1$. However, it is readily demonstrated that

$$R(n, N) = R(|n|, N) \quad \dots (17)$$

for $-(N-1) \leq n \leq 0$.

These latter results can be combined to give

$$R(n, N) = S \Delta t \sum_{j=1}^{N-|n|} h_j h_{|n|+j} \quad \dots (18)$$

for $-(N-1) \leq n \leq (N-1)$.

Using the h coefficients already defined for $\lambda = \frac{1}{2}$ yields

$$R(n, N) = 2 S K^2 \sum_{j=1}^{N-|n|} \frac{1}{\sqrt{j(j+|n|)}}$$

or expressed differently

$$R(n, N) = 2 S K^2 R^1(n, N), \quad \dots (19)$$

where

$$R^1(n, N) = \sum_{j=1}^{N-|n|} \frac{1}{\sqrt{j(j+|n|)}}$$

4. CALCULATION OF SPECTRAL POWER DENSITY

For a continuous process, the spectral power density $\phi(f)$, corresponding to an autocorrelation function $R(\tau)$, is given by

$$\phi(f) = \int_{-\infty}^{\infty} R(\tau) e^{-j2\pi f\tau} d\tau. \quad \dots (20)$$

It is assumed that for the discrete system being considered, the spectral power density can be similarly calculated using the relationship

$$\phi(f) = 2 SK^2 \Delta t \sum_{n=-(N-1)}^{(N-1)} R^1(n, N) e^{-j2\pi f n \Delta t} \quad \dots (21)$$

This may also be represented as

$$\phi(f) = 2 SK^2 \Delta t \sum_{n=-(N-1)}^{(N-1)} R^1(n, N) \cos(2\pi f n \Delta t), \quad \dots (22)$$

since $R^1(n, N)$ is an even function.

For the actual calculations a modified form of equation (23) was used i.e.

$$\frac{\phi(f)}{2NSK^2 \Delta t} = \frac{1}{M-1} \sum_{m=1}^{2M-2} R^1((m - M+1), M) e^{-j2\pi f m \Delta t}, \quad \dots (23)$$

where $M = N + 1$.

The lower limit of m was changed from 0 to 1, since $R^1(n, M)$ is symmetrical about $R^1(0, M)$, the number of coefficients calculated are $(2M-1)$. For efficient use of the Fast Fourier Transform (FFT) the $R^1(-(M-1), M)$ term is discarded so the number of coefficients is a power of 2. This was found to have negligible effect on the results.

Equation (23) has been evaluated using time decimation FFT techniques for various values of N . Sample results are shown in the log-log plots of Figures 2, 3, 4 and 5. The points in these plots appear to lie within two distinct lines, which are the envelopes of the oscillations explained

in Section 5 and illustrated in Figure 6. The continuous straight line shown in each figure is the least squares fitted line with the line equation shown on the figure. The frequency exponent α is independent of Δt but is a function of N . The α values calculated using the least squares fits are shown in Table 1.

TABLE 1
COMPARISON OF SPECTRAL POWER RESPONSE
EXPONENTS $(f)^{-\alpha}$ FOR VARIATION OF N

N	α
256	1.23
512	1.23
1024	1.22
2048	1.22

The fit was made over the frequency range $\frac{1}{2N} \leq f \Delta t \leq \frac{1}{2}$. It is evident that α exceeds unity, but decreases as N increases.

Equation (23) can be used directly to evaluate the low frequency ($f \rightarrow 0$) power spectrum by noting that

$$\sum_{n=-(N-1)}^{N-1} R^1(n, N) \approx 4N, \quad \dots (24)$$

as suggested in Appendix II. This means that

$$\lim_{f \rightarrow 0} \frac{\phi(f)}{2NSK^2 \Delta t} = 4. \quad \dots (25)$$

This low frequency asymptote is plotted on Figures 2, 3, 4, 5 and 6 as well as the high frequency asymptote for the continuous case derived in Appendix III.

$$\text{As } f \rightarrow \infty \quad \frac{\phi(f)}{2NSK^2\Delta t} \sim \frac{1}{2N(f\Delta t)} \quad \dots (26)$$

5. POWER SPECTRUM RIPPLE

As is evident in Figure 6 and suggested in Figures 2, 3, 4 and 5 by the envelopes, the spectra have oscillatory characteristics superimposed on a smoothly varying component. It is suggested by Ternan (6) that such behaviour is directly attributable to the truncation of the impulse response. In Appendix III this proposal is investigated for the case of a continuous system with an abruptly truncated impulse response. It is shown that the oscillatory component has a repetition "frequency" in the frequency domain of $N\Delta t$ and that the ratio of oscillatory amplitude to basic component decreases with frequency according to an f^{-2} law. This trend is also expected with the discrete model being discussed.

Figure 6 shows an evaluation of the normalised version of equation (22).

$$\text{i.e.} \quad \frac{\phi(f)}{2NSK^2\Delta t} = \frac{1}{N} \sum_{n=-(N-1)}^{(N-1)} R^1(n, N) \cos(2\pi f n \Delta t) \quad \dots (27)$$

This shows the fluctuations in the low frequency end of the spectrum, and is a direct comparison with Figure 4.

The value of $\alpha = 1.21$ shown in Figure 6 is not as reliable as that in Figure 4 because of the small number of points used in the least squares fit.

6. CONCLUSION

Spectral analysis of the process

$$y_j = \sqrt{2K}(\Delta t)^{\frac{1}{2}} \sum_{i=j-N}^{j-1} \frac{x_i}{\sqrt{j-i}},$$

operating on samples x_i from a white noise generator of uniform power spectral density S has been undertaken. This process was developed to calculate digital values of quasi-flicker noise for use in a computer simulation.

For small values of N the spectral power density has a characteristic of $f^{-\alpha}$ where α gradually decreases from 1.23 as N increases above 250.

A similar problem was encountered by Barnes and Allen with their original "flicker" noise generator which had an $f^{-0.8}$ rather than an f^{-1} characteristic. As in their case, a more appropriate value of λ could be determined to give an f^{-1} variation for small values of N .

APPENDIX I

DERIVATION OF IMPULSE RESPONSE $H(t)$

Consider a linear network with transfer function

$$G(f) = K(jf)^{-\lambda}, \quad -\infty < f < \infty \quad \dots (A1)$$

This may be re-expressed as

$$\left. \begin{aligned} G(f) &= K|f|^{-\lambda} e^{-\frac{j\pi\lambda}{2}}, \quad f > 0 \\ G(f) &= K|f|^{-\lambda} e^{\frac{j\pi\lambda}{2}}, \quad f < 0 \end{aligned} \right\} \quad \dots (A2)$$

The network's impulse response may be written

$$H(t) = \int_{-\infty}^0 G(f) e^{j2\pi ft} df + \int_0^{\infty} G(f) e^{j2\pi ft} df \quad \dots (A3)$$

where the first term applies to negative values of f and the second term applies to positive f . On substituting the appropriate expressions from (A2) into (A3) it follows that

$$H(t) = 2K \int_0^{\infty} f^{-\lambda} \cos\left(2\pi ft - \frac{\pi\lambda}{2}\right) df \quad \dots (A4)$$

Using the substitution $x = 2\pi ft$ gives

$$H(t) = 2K(2\pi t)^{\lambda-1} \int_0^{\infty} x^{-\lambda} \cos\left(x - \frac{\pi\lambda}{2}\right) dx, \quad t > 0 \quad \dots (A5)$$

$$\text{and } H(t) = 2K(-2\pi t)^{\lambda-1} \int_0^{\infty} x^{-\lambda} \cos\left(x + \frac{\pi\lambda}{2}\right) dx, \quad t < 0 \quad \dots (A6)$$

Now it may be shown, using standard contour integration methods, that

$$\int_0^{\infty} x^{-\lambda} \cos\left(x - \frac{\pi\lambda}{2}\right) dx = \sin(\pi\lambda) \Gamma(1-\lambda), \quad \dots (A7)$$

$$\text{and } \int_0^{\infty} x^{-\lambda} \cos\left(x + \frac{\pi\lambda}{2}\right) dx = 0 \quad \dots (A8)$$

provided $0 < \lambda < 1$.

Substituting the expressions from (A7) and (A8) into (A5) and (A6) respectively, gives

$$\begin{aligned} H(t) &= 2K \sin(\pi\lambda) \Gamma(1-\lambda) (2\pi t)^{\lambda-1}, \quad t > 0 \\ \text{and } H(t) &= 0, \quad t < 0 \end{aligned} \quad \dots (A9)$$

For the particular condition $\lambda = \frac{1}{2}$, and noting that $\Gamma(\frac{1}{2}) = \sqrt{\pi}$, it follows that

$$H(t) = \sqrt{2} K t^{-\frac{1}{2}} \quad \dots (A10)$$

APPENDIX II

EVALUATION OF $\epsilon R^1(n, N)$

As shown in the text, the low frequency spectral power density is controlled by the value of

$$A = \sum_{n=-(N-1)}^{N-1} R^1(n, N) ,$$

$$\text{where } R^1(n, N) = \sum_{j=1}^{N-|n|} \frac{1}{\sqrt{j(j+|n|)}} , \quad -(N-1) \leq n \leq N-1$$

No standard formula appears to be available for the above summation, although, Macdonald (8) and Harper (9) have both suggested relationships of the form

$$A \sim 4N + C_1(4N^{\frac{1}{2}} + N^{-\frac{1}{2}}) + C_2N^{-1} + C_3 ,$$

where C_1 , C_2 and C_3 are constants, for evaluating the double summation.

However, on the assumption that the low frequency power density for the discrete case and the continuous case as analysed in Appendix III should be closely the same, then it is expected that $A \sim 4N$ as $N \rightarrow \infty$. This proposition was tested numerically for various N values giving the results below :

N	A	4N
10	25.2	40
50	162.6	200
100	345.6	400
500	1873.4	2000
1000	3819.9	4000

It is concluded that the proposition, $A = 4N$, is accurate within 5% for $N > 1000$.

APPENDIX III

POWER SPECTRUM FOR A CONTINUOUS SYSTEM

Consider a linear system with impulse response

$$H(t) = \sqrt{2K}t^{-\frac{1}{2}}, \quad 0 \leq t \leq T \quad \dots (A6)$$

Its transfer function is

$$G(f) = \int_{-\infty}^{\infty} H(u)e^{-j2\pi fu} du, \quad \dots (A7)$$

which on substitution of (A6) gives

$$G(f) = \frac{K}{\sqrt{\pi f}} \int_0^{2\pi fT} u^{-\frac{1}{2}} e^{-ju} du \quad \dots (A8)$$

LOW FREQUENCY BEHAVIOUR

The function in (A8) can be written in series form,

$$G(f) = \frac{K}{\sqrt{\pi f}} \sum_{n=0}^{\infty} \frac{(-j)^n}{n!} \frac{1}{(n + \frac{1}{2})} (2\pi fT)^{n+\frac{1}{2}}, \quad \dots (A9)$$

from which it follows that

$$G(0) = 2\sqrt{2T} K \quad \dots (A10)$$

The output spectral power density for a discrete process at zero frequency is then

$$\begin{aligned} \phi(0) &= |G(0)|^2 S \\ &= 8N\Delta t K^2 S, \end{aligned} \quad \dots (A11)$$

where T, the duration of the impulse response, is replaced by NΔt and S is the input power density.

HIGH FREQUENCY BEHAVIOUR

$G(f)$, from (A8), can be written

$$G(f) = \frac{K}{\sqrt{\pi f}} \int_0^{\infty} u^{-1/2} e^{-ju} du - \frac{K}{\sqrt{\pi f}} \int_{2fT}^{\infty} u^{-1/2} e^{-ju} du,$$

which, using results from Copson (7), gives

$$G(f) = \frac{K}{\sqrt{f}} e^{-j\pi/4} - \frac{K}{\pi f \sqrt{2T}} [X - jY] e^{-j2\pi fT} \quad \dots (A12)$$

where, if (fT) is large and positive,

$$\left. \begin{aligned} X &\sim \frac{1}{4\pi fT} - \frac{1.3.5}{(4\pi fT)^3} + \frac{1.3.5.7.9}{(4\pi fT)^5} + \dots \\ \text{and } Y &\sim 1 - \frac{1.3}{(4\pi fT)^2} + \frac{1.3.5.7}{(4\pi fT)^4} - \dots \end{aligned} \right\} \quad \dots (A13)$$

If we consider only

$$2\pi fT \geq 5,$$

then $Y \approx 1$ and $X \approx 0$.

The output spectral power density $\phi(f)$ is given by

$$\phi(f) = SG(f) G^*(f). \quad \dots (A14)$$

Substituting (A12) in (A14) with the restricted values of X and Y gives the normalised power spectrum

$$\begin{aligned} \frac{\phi(f)}{2NSK^2\Delta t} &\sim \frac{1}{2N(f\Delta t)} + \frac{1}{4\pi^2 N^2 (f\Delta t)^2} - \\ &\frac{1}{2\pi^2 N^3} \frac{1}{(f\Delta t)^{3/2}} \sin \left[\frac{\pi}{4} - 2\pi N(f\Delta t) \right], \quad \dots (A15) \end{aligned}$$

where as before $N\Delta t = T$.

The first term on the right hand side represents the required flicker term i.e. f^{-1} . The second term, which varies with f^{-2} , and the third term, which oscillates with an amplitude proportional to $f^{-3/2}$, can be regarded as error terms. The oscillatory term repeats at intervals of $\frac{1}{N}$ along the $(f\Delta t)$ abscissa.

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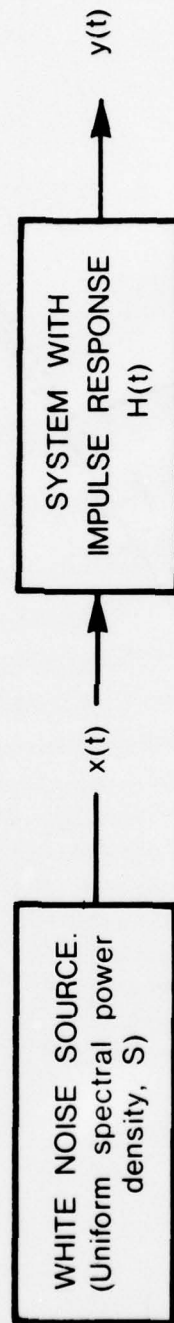


FIG. 1 - Block diagram showing the input of white noise $x(t)$ to a filter with impulse response $H(t)$.

SPECTRAL POWER DENSITY N= 256

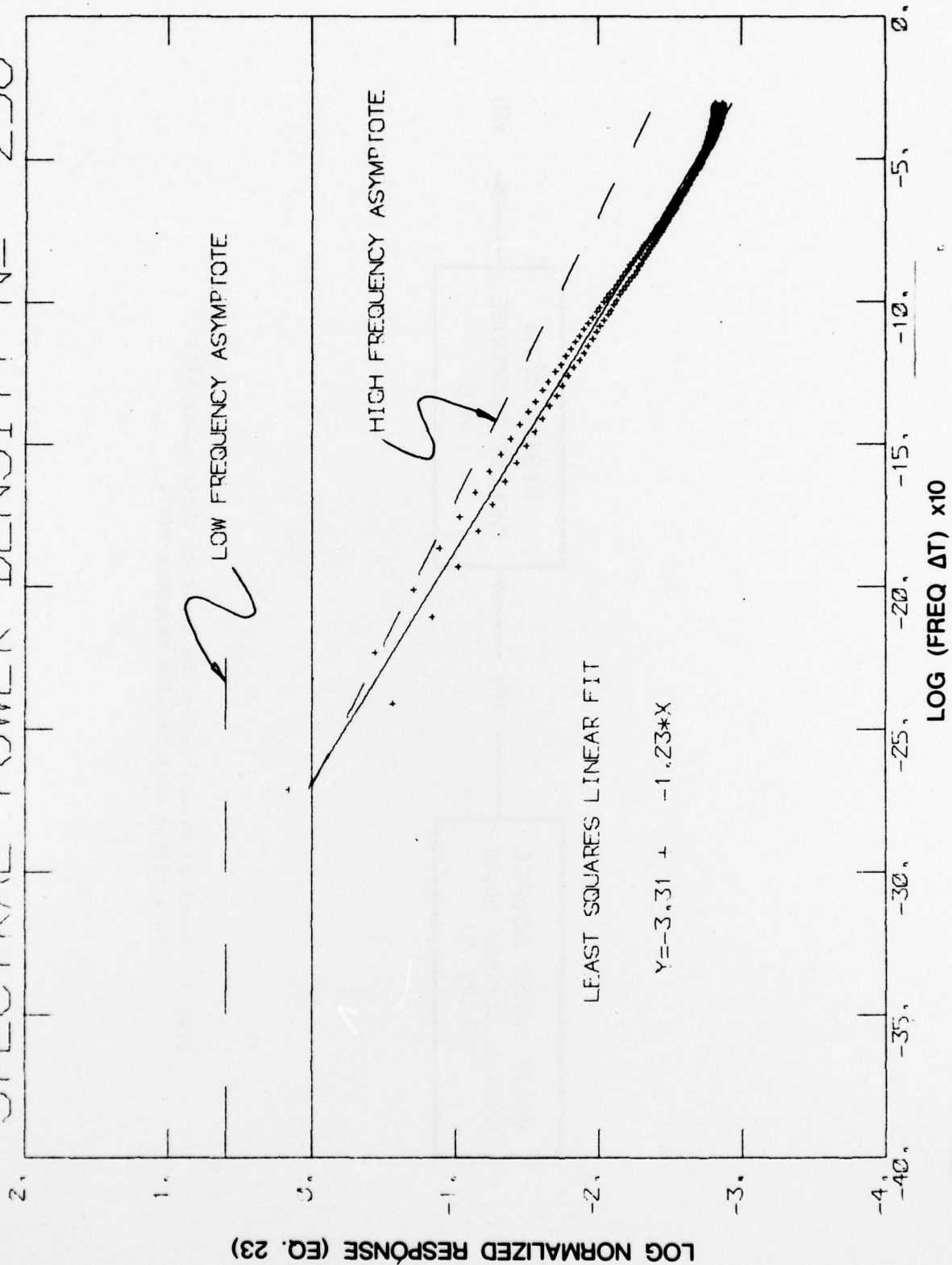


FIG. 2

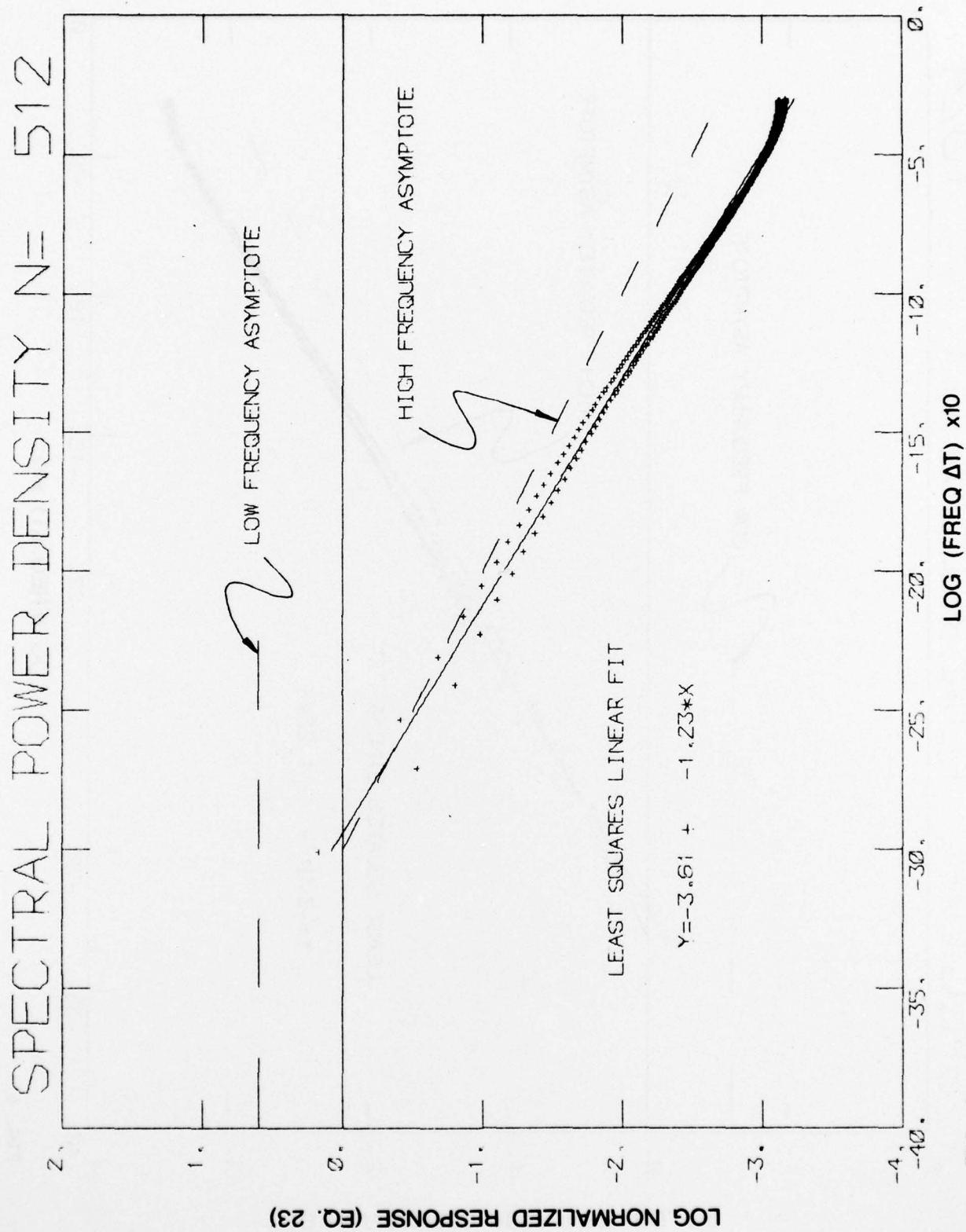


FIG. 3

SPECTRAL POWER DENSITY N= 1024

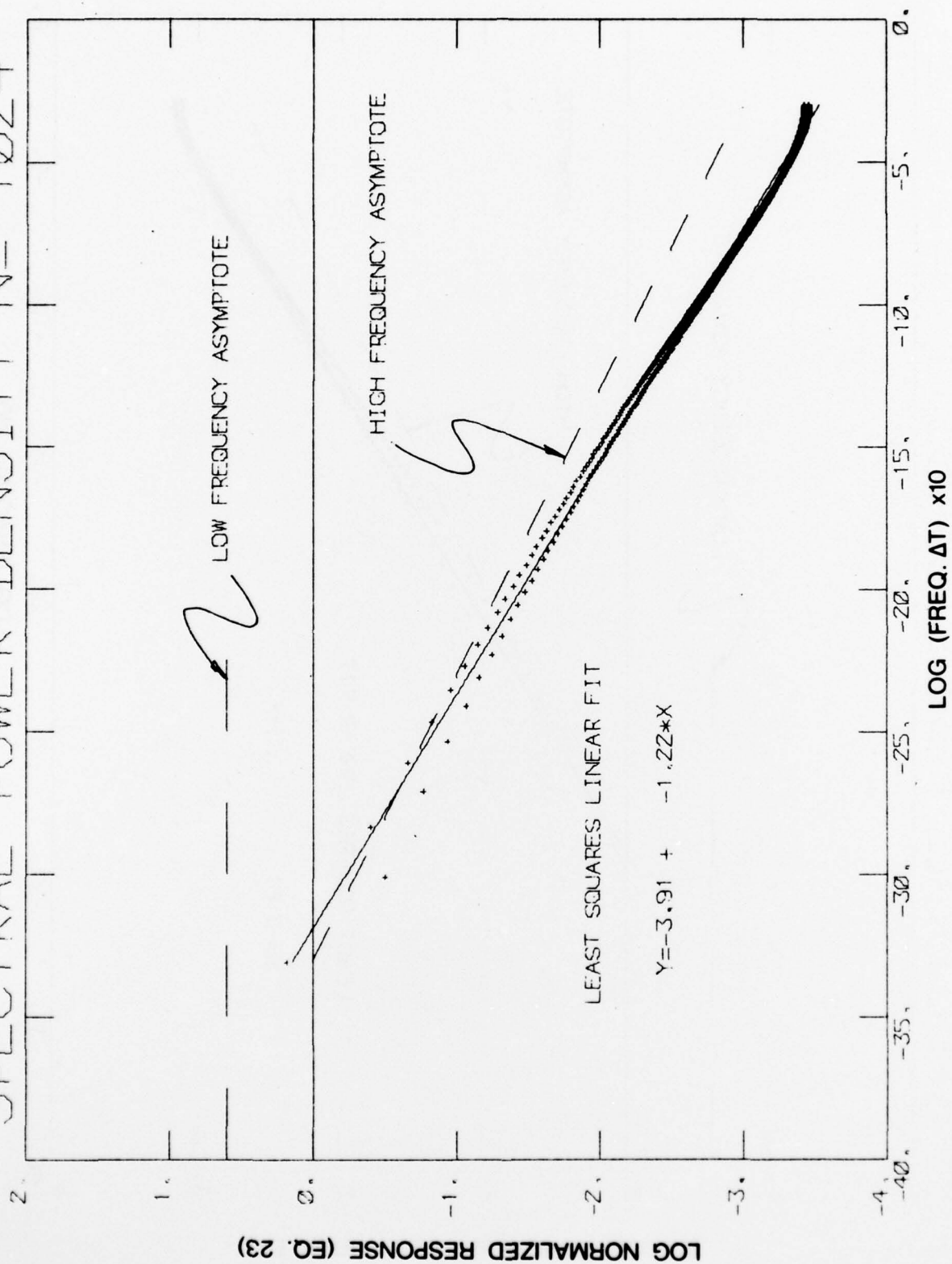


FIG. 4

SPECTRAL POWER DENSITY N= 2048

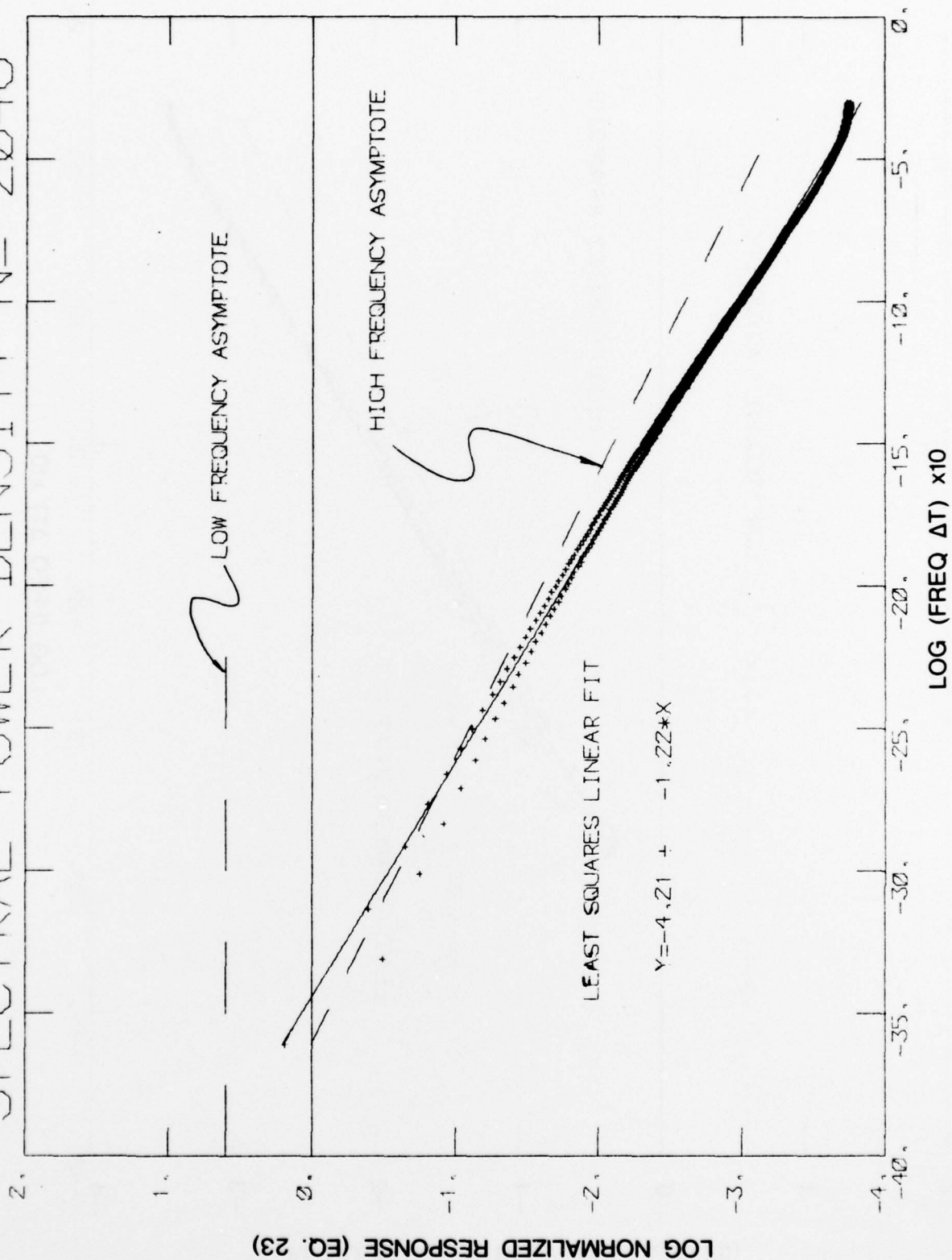


FIG. 5

SPECTRAL POWER DENSITY N=1024

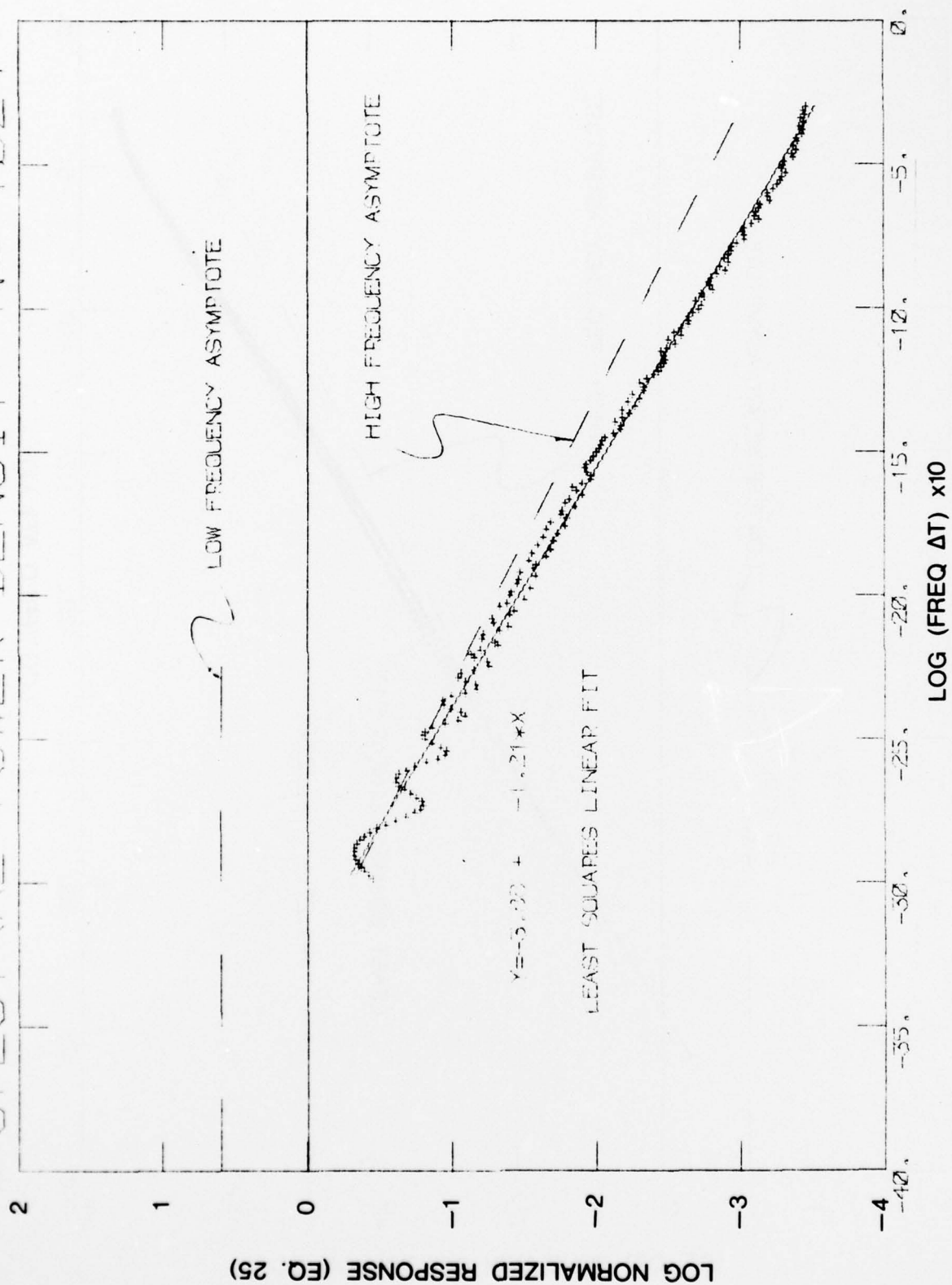


FIG. 6

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